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UNSTEADY-STATE CONJUGATED HEAT TRANSFER BETWEEN A SEMI-INFINITE SURFACE AND INCOMING FLOW OF A COMPRESSIBLE FLUID—I. REDUCTION TO THE INTEGRAL RELATION

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Abstract—The analytical solution of the conjugated unsteady heat transfer problem in a semi-infinite plate with the sources in the form of the generalized power series is obtained in the space of the generalized functions (in the Sobolev–Schwartz sense).

The dimensionless parameter $B = [(1/Re).(a_s/Ca_\infty)]$ having physical meaning is found which allows consideration of the problem as the unsteady or quasi-stationary one depending on the "body–liquid" pair.

NOMENCLATURE

x, y, rectangular Cartesian coordinate system related to body;
u, v, liquid velocity in x, y-directions, respectively;
T, temperature;
q, heat flux from unit surface per unit time;
λ, thermal conductivity;
a, thermal diffusivity;
c, heat capacity;
C, constant from Chapman–Rubesin's law;
ρ, density;
μ, coefficient of dynamic viscosity;
ν, coefficient of kinematic viscosity;
Pr, Re, Nu, M, Prandtl, Reynolds, Nusselt, Mach numbers, respectively;
L, H, characteristic dimension and thickness of a body in a flow, respectively.

Subscripts

∞, s, w, refer to the incoming liquid flow, body in a flow and its surface. The symbols without subscripts refer to the liquid in the boundary layer.

1. INTRODUCTION

THE SOLUTION of the problem on convective heat transfer between a body and a liquid is reduced to finding temperature fields in a body and a liquid. Due to mathematical difficulties which arise in solving such problems two approaches are established.

In case of a small temperature drop between a body and a liquid, only a temperature field of a body was determined. In this case the heat flux from a body into a liquid was prescribed by the Newtonian convective heat transfer law: $q = \alpha(T_w - T_\infty)$, where α is the known heat transfer coefficient.

The temperature field was found by solving the heat conduction equation and the heat flux, by the above formula.

At considerable temperature drops between a body and an incoming flow the attention was concentrated on finding a temperature field of a liquid. Upon solving the problem the Nusselt number was calculated by the formula

$$Nu = -\frac{L}{T_w - T_\infty}\cdot\left(\frac{\partial T}{\partial n}\right)_{n=0}. \quad (1)$$

In such problems the body surface temperature or the heat flux from a body surface was considered to be beforehand prescribed.

In the first approach the thermophysical properties of a liquid were not taken into account. The second approach does not take into account the thermophysical properties of a body. For heat transfer when the surface is essentially non-isothermal this leads to the results which are not valid from the viewpoint of physical grounds. Therefore, a temperature field in a liquid and that in a body should be calculated simultaneously [1, 2].

In [3] it is shown that for intense steady-state heat transfer of a thin plate in a parallel flow of a compressible liquid it is impossible to predetermine the *a priori* value of the body surface temperature T_w in a flow. In [3] T_w was considered to be the unknown function and was found by simultaneous solution of the equation for convective heat transfer in a boundary layer and of the equation of heat conduction in a body (a conjugated problem). The heat flux and temperature at the interface were considered to be continuous, i.e.

$$T|_{y=+0} = T_s|_{y=-0},\qquad \left(\lambda\frac{\partial T}{\partial y}\right)\bigg|_{y=+0} = \lambda_s\frac{\partial T_s}{\partial y}\bigg|_{y=-0}.$$

2. STATEMENT OF PROBLEM

A parallel flow of a compressible liquid with a constant velocity U_∞ flows onto the external surface of a plate ($y = 0$). It is assumed that $10 < Re < Re_{cr}$. The boundary layer above the plate then becomes laminar.

The heat sources with a density $Q_0(x, y, t)$ distributed in a plate and dependent on the coordinates are introduced from some moment $t = 0$ and given in the form

$$Q_0 = 1(t)\sum_{m=0}^{\infty} Q_{0,m}(y, t)\,x^{\rho+m},\quad (0 \leqslant \rho < 1)$$

and the heat flux at the lower plate surface $y = -H$ has the form

$$F_0(x, t) = 1(t)\sum_{m=0}^{\infty} F_{0,m}(t)\,x^{\rho+m},$$

where $1(t)$ is the Heaviside function. We think that the functions $1(t)Q_{0,m}(y, t)$ at a fixed value of y and functions $1(t)F_{0,m}(t)$ are the infinitely differentiable functions with a restricted carrier, i.e. from a space $K(R\colon -\infty < t < +\infty)$. The solution will be sought in the space K' of the generalized functions (distributions) in the Sobolev–Schwartz sense [4]. Thus, all operations over t should be understood in the sense of operations in the space K'. Furthermore, we shall bear in mind that all time-dependent functions contain the Heaviside function which ensures the conditions in the region $t < 0$ to be performed. It is omitted to make the writing more short. Note that the derivatives over t, in the meaning of the generalized functions at $t > 0$ and $t < 0$ coincide with the derivatives in common sense [4], and the solution at $t > 0$ with the classical solution. Use the usual assumptions: $c_p = \text{const}$, $Pr = \text{const}$, $\rho\mu = \text{const}$. The temperature distribution $T_e(x, y)$ in the boundary layer at $t < 0$ is known and satisfies the conditions of the thermal insulation of a plate [5].

The conjugated heat transfer problem is then written thus:

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \tag{2}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0, \tag{3}$$

$$c_p\rho\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2, \tag{4}$$

$$c_s\rho_s\frac{\partial T_s}{\partial t} = \lambda_s\left(\frac{\partial^2 T_s}{\partial x^2} + \frac{\partial^2 T_s}{\partial y^2}\right) + Q_0(x, y, t), \tag{5}$$

$$u\big|_{\substack{y=0\\x>0}} = v\big|_{\substack{y=0\\x>0}}, \quad u\big|_{y=\infty} = U_\infty,$$
$$v\bigg|_{\substack{y=0\\x<0}} = \frac{\partial u}{\partial y}\bigg|_{\substack{y=0\\x<0}} = 0, \quad (6)$$

$$\lambda\frac{\partial T}{\partial y}\bigg|_{\substack{y=0\\x<0}} = 0, \quad T\big|_{y=\infty} = T_\infty, \quad (7)$$

$$T\big|_{t<0} = T_e(x,y), \quad \frac{\partial T_e}{\partial y}\bigg|_{y=0} = 0, \quad T_e\big|_{y=\infty} = T_\infty. \quad (8)$$

The boundary and initial conditions in a body are:

$$\lambda_s\frac{\partial T_s}{\partial x}\bigg|_{x=0} = 0, \quad \lambda_s\frac{\partial T_s}{\partial x}\bigg|_{x=\infty} = 0; \quad (9)$$

$$\lambda_s\frac{\partial T_s}{\partial y}\bigg|_{y=-H} = F_0(x,t), \quad T_s\big|_{t<0} = T_\infty N(0). \quad (10)$$

(The number $N(0)$ will be defined below.)

The conditions of conjugation are

$$T\big|_{\substack{y=0\\x>0}} = T_s\big|_{\substack{y=0\\x>0}},$$
$$\lambda_w\frac{\partial T}{\partial y}\bigg|_{\substack{y=0\\x>0}} = \lambda_s\frac{\partial T_s}{\partial y}\bigg|_{\substack{y=0\\x>0}}. \quad (11)$$

3. TRANSITION TO TIME-DEPENDENT COORDINATE SYSTEM

Following Moore [6] introduce the stream function $\psi(x,y,t)$, and in case of a liquid pass to new independent variables†

$$X = x, \quad Y = \int_0^y \frac{\rho(x,u,t)}{\rho_\infty}\,du, \quad t^* = t. \quad (12)$$

Then, using Chapman–Rubesin's law [9] for the relation of viscosity versus temperature $\mu/\mu_\infty = CT/T_\infty$ we have

$$\frac{\partial^2\psi}{\partial t^*\,\partial Y} + \frac{\partial\psi}{\partial Y}\cdot\frac{\partial^2\psi}{\partial X\,\partial Y} - \frac{\partial\psi}{\partial X}\frac{\partial^2\psi}{\partial Y^2} = Cv_\infty\frac{\partial^3\psi}{\partial Y^3}, \quad (13)$$

$$\frac{\partial\psi}{\partial Y}\bigg|_{\substack{Y=0\\X>0}} = \frac{\partial\psi}{\partial X}\bigg|_{\substack{Y=0\\X>0}} = 0, \quad \frac{\partial\psi}{\partial Y}\bigg|_{Y=\infty} = U_\infty, \quad (14)$$

$$\frac{\partial\psi}{\partial Y}\bigg|_{\substack{Y=0\\X<0}} = \frac{\partial^2\psi}{\partial Y^2}\bigg|_{\substack{Y=0\\X<0}} = 0, \quad (15)$$

$$\frac{\partial\bar{T}}{\partial t^*} + \frac{\partial\psi}{\partial Y}\frac{\partial\bar{T}}{\partial X} - \frac{\partial\psi}{\partial X}\frac{\partial\bar{T}}{\partial Y} = Ca_\infty\frac{\partial^2\bar{T}}{\partial Y^2} + \frac{Cv_\infty}{c_p}\left(\frac{\partial^2\psi}{\partial Y^2}\right)^2, \quad (16)$$

where

$$\bar{T} = \bar{T}(X,Y,t^*) = T(X, y(X,Y,t^*), t^*),$$

$y,(X, Y, t^*)$ is the inverse function for the integral in (12).

$$C\lambda_\infty\frac{\partial\bar{T}}{\partial Y}\bigg|_{\substack{Y=0\\X<0}} = 0. \quad (17)$$

From conditions (7), (8) in the new variables we have

$$\bar{T}(X,Y,t^*)\big|_{Y\to\infty} \to T_\infty \quad (18)$$

$$\bar{T}(X,Y,t^*)\big|_{t^*<0} = \bar{T}_e(X,Y). \quad (19)$$

The conjugation conditions are

$$\bar{T}\bigg|_{\substack{Y=0\\X>0}} = T_s\bigg|_{\substack{y=0\\X>0}}, \; C\lambda_\infty\frac{\partial\bar{T}}{\partial Y}\bigg|_{\substack{Y=0\\X>0}} = \lambda_s\frac{\partial T_s}{\partial y}\bigg|_{\substack{y=0\\X>0}}, \quad (20)$$

Conditions (9), (10) and equation (5) did not vary.

The transition (6), (7), respectively, in (14), (18) needs to be proved to some extent, i.e. it is necessary to show that

$$Y = Y(x,y,t) \to \infty \text{ at } y\to\infty. \quad (20')$$

† Moore–Stewartson's transformation [6, 7] develops the idea on the general transformation of the coordinates, proposed by A. A. Dorodnitsyn [8].

‡ Prandtl (13) and energy (16) equations agree with the appropriate equations for an incompressible liquid involving kinematic viscosity Cv_∞ and thermal diffusivity coefficient $C\alpha_\infty$.

Really†

$$\lim_{y\to\infty} Y/y = \lim_{y\to\infty} \frac{\partial Y/\partial y}{1} = \lim_{y\to\infty} \rho/\rho_\infty = 1. \quad (21)$$

Hence, statement (20′) follows.

Note that in the transformation plane X, Y the hydrodynamic problem (13)–(15) does not depend upon thermal conjugated problem (5), (9), (10), (16)–(20). Therefore, $\psi(X,Y)$, being a solution to the steady-state problem at $t^* < 0$, will also describe the flow at $t^* > 0$, i.e. a change in thermal conditions will influence only $Y = Y(x, y, t)$.

Thus, thermal effects in the plane X, Y are eliminated, the hydrodynamic problem is a steady-state one and the thermal problem is an unsteady-state one.

4. TRANSITION TO PARABOLIC COORDINATES

In the boundary layer equations the parabolic coordinates are used

$$X = \bar{\xi}^2 - \bar{\eta}^2,\ Y = 2\bar{\xi}\bar{\eta},\ t^* = \bar{\tau}.$$

Pass to the dimensionless "hydrodynamic" coordinates, let $Re \to \infty$ and then pass to "thermal" coordinates

$$\bar{\xi}, \bar{\eta}, \bar{\tau} \to \xi = \frac{1}{\sqrt{L}}\bar{\xi},\ \eta = \left(\frac{2U_\infty}{Cv_\infty}\right)^{\frac{1}{2}}\bar{\eta},$$

$$\tau' = \frac{2U_\infty}{L}\bar{\tau} \to \xi, \eta, \tau = \frac{a_s}{L^2}\bar{\tau}.$$

Pass to the dimensionless variables within a body

$$z = \frac{X}{L}(X = x), \quad \zeta = \frac{y}{L}, \quad \tau = \frac{a_s}{L^2}\bar{\tau}.$$

Then, the stated problem (5), (9), (10), (13)–(20) assumes the form:

(a) hydrodynamic problem (13)–(15)

$$f'''(\eta) + f(\eta)f''(\eta) = 0, \quad f(0) = f'(0) = 0,$$

$$f'(\infty) = 1,$$

where

$$\psi = (2Cv_\infty LU_\infty)^{\frac{1}{2}}\xi f(\eta)$$

(b) thermal conjugated problem (5), (9), (10), (16)–(20)

$$\frac{\partial^2 T^*}{\partial \eta^2} + Prf\frac{\partial T^*}{\partial \eta} - Prf'\xi\frac{\partial T^*}{\partial \xi}$$

$$- B\xi^2\frac{\partial T^*}{\partial \tau} = -Pr(\gamma - 1)M_\infty^2(f'')^2, \qquad (22)$$

where

$$B = \frac{2a_sPr}{Cv_\infty Re} = \frac{2}{Re}\frac{a_s}{Ca_\infty}, \quad T^* = \frac{T}{T_\infty}. \qquad (23)$$

$$\left.\frac{\partial T^*}{\partial \xi}\right|_{\xi=0} = 0, \qquad (24)$$

$$T^*(\xi, \eta, \tau) \to 1 \quad \text{at} \quad \eta \to \infty \qquad (25)$$

$$T^*(\xi, \eta, \tau)|_{\tau<0} = T_e(\xi, \eta),$$

$$\left.\frac{\partial T_e^*}{\partial \eta}\right|_{\eta=0} = 0, \quad T_e^*\Big|_{\substack{\eta=\infty\\ \xi=\infty}} = 1. \quad (26)$$

Before solving the problem, consider the physical meaning of the parameter B.

5. PHYSICAL MEANING OF THE PARAMETER B

The parameter B has the following physical meaning. Let some amount of heat release at the point A of the plate. At the point C of the plate located from the point A at a distance L the effect of heat impulse at the point will appear in time $t_s \sim L^2/a_s$. The time t, in which heat impulse at the point A of the plate appears on the boundary of the boundary layer (at the point N) has the order $t \sim \delta^2/a_{\text{eff}}$, where $\delta \sim L/\sqrt{(Re)}$, $a_{\text{eff}} \sim Ca_\infty$, a_{eff} is the thermal diffusivity of a gas near a plate. Then we have $t/t_s \sim a_s/ReCa_\infty$. Therefore $B \sim t/t_s$†. From (23) it is seen that the parameter B decreases with

† Although the behaviour of the function $Y(x, y, t)$ at $y \to \infty$ is beforehand unknown, the result from [10] allows the function ratio to be replaced by the ratio of their derivatives.

† The meaning of B remain the same in case of incompressible fluid flow past a plate either.

an increase in Re since $2a_s/Ca_\infty$ is fixed for each body–liquid pair.

At $B \ll 1$ the term $B\xi^2(\partial T^*/\partial\tau)$ may be neglected in energy equation (22). At the same time the heat conduction equation, conjugation conditions, etc do not change. The energy equation contains time only as a parameter.

Thus, the conjugated unsteady-state heat transfer problem (when unsteadiness is caused by the sources in a body) at $B \ll 1$ may be considered as quasi-stationary in the boundary layer and unsteady in the plate. The case $B \ll 1 (t \ll t_s)$ means physically, that complete heating of a boundary layer occurs during the period when heat impulse covers a distance in the plate, which is much less L.

Thus, conjugated unsteady heat transfer problems (when unsteadiness is caused by the sources in the plate) may be divided into two classes, if the number $E = a_s/Ca_\infty$ is introduced ($E = a_s/a_\infty$ for incompressible liquid). At $Re \leqslant E$ the problem may be solved as the unsteady one both in the boundary layer and within the body, and at $Re \gg E$, as the unsteady one within the body and the quasi-stationary one in the boundary layer.† The number E depends on the thermophysical properties of a plate and the properties of the fluid medium. The values of E for different body–liquid pairs are given in Table 1.

6. PARTIAL SOLUTION OF ENERGY EQUATION

The solution of a dimensionless heat conduction equation is sought in the form:

$$T_s^* = \Theta_s(z, \zeta, \tau) + N(0).$$

Then we have

$$\frac{\partial \Theta_s}{\partial \tau} = \frac{\partial^2 \Theta_s}{\partial z^2} + \frac{\partial^2 \Theta_s}{\partial \zeta^2} + Q(z, \zeta, \tau), \tag{27}$$

where $Q \overset{\text{def}}{=} \sum_{k=0}^{\infty} Q_K(\zeta, \tau) z^{\rho+k}$,

† In case of the sources rapidly varying with time ($\tau_0 = t_0/(L^2/a_s) \ll 1$ is the characteristic time of a change in the heat source) the derivative $\partial T^*/\partial\tau$ may be considerable, since it has the order of $\sim T^*/\tau_0$. Then $B\,\partial T^*/\partial\tau \sim B/\tau_0\, T^*$ and $E = a_s/Ca_\infty\tau_0$. Hence, E in the case under consideration also depends on the sources.

$$\left.\frac{\partial \Theta_s}{\partial z}\right|_{z=0} = 0, \tag{28}$$

$$\left.\frac{\partial \Theta_s}{\partial z}\right|_{z=\infty} = 0, \tag{29}$$

$$\left.\frac{\partial \Theta_s}{\partial \zeta}\right|_{\zeta=-h} = F(z, \tau) \overset{\text{def}}{=} \sum_{k=0}^{\infty} F_k(\tau) z^{\rho+k}, \tag{30}$$

$$\Theta_s(z, \zeta, \tau) = 0 \quad \text{at} \quad \tau < 0, \tag{31}$$

$$h = \frac{H}{L}, \quad F(z, \tau) = \frac{L}{\lambda_s T_\infty} F_0\left(Lz, \frac{L^2}{a_s}\tau\right),$$

$$Q(z, \zeta, \tau) = \frac{L^2}{\lambda_s} Q_0\left(Lz, L\zeta, \frac{L^2}{a_s}\tau\right). \tag{32}$$

The conjugated conditions are

$$T^*|_{\eta=0} = \Theta_s|_{\substack{\zeta=0\\ z=\xi^2}} + N(0) \tag{33}$$

$$Mh\frac{1}{\xi}\left.\frac{\partial T^*}{\partial \eta}\right|_{\eta=0} = \left.\frac{\partial \Theta_s}{\partial \zeta}\right|_{\substack{\zeta=0\\ z=\xi^2}}, \tag{34}$$

where

$$M = \frac{\sqrt{(c)}\,\lambda_\infty}{h\lambda_s}\left(\frac{Re}{2}\right)^{\frac{1}{2}}.$$

The solution of energy equation (22) is sought in the form:

$$T^*(\xi, \eta, \tau) = \Theta(\xi, \eta, \tau) + T_1^*(\xi, \eta, \tau), \tag{35}$$

where $\Theta(\xi, \eta, \tau)$ is the general solution of the appropriate uniform equation, $T_1^*(\xi, \eta, \tau)$ is the partial solution of non-uniform equation (22).

Using the separation-of-variables technique, the partial solution T_1^* is sought in the form

$$T_1^* = S(\xi) + N(\eta) + P(\tau) = C_1 \ln \xi + N(\eta) + C_2\tau.$$

The demand for a temperature to be limited gives

$$T_1^*(\xi, \eta, \tau) = N(\eta), \tag{36}$$

where $N(\eta)$ satisfies the equation

$$\frac{d^2N}{d\eta^2} + Prf\frac{dN}{d\eta} = -Pr(\gamma - 1)M_\infty^2(f'')^2. \tag{37}$$

Table 1

		$t°$C	Aluminium	Iron	Silver	Copper	M_∞
Water	E	+20	6×10^2	$1{\cdot}5 \times 10^2$	$11{\cdot}8 \times 10^2$	$7{\cdot}8 \times 10^2$	–
	E	−50	6·8	1·7	13·3	8·8	1
Air	E	0	4·7	1·1	8·9	6·0	1
	E	+50	3·4	0·7	6·3	4·4	1
	E	−50	7·4	1·6	13·9	9·6	2
	E	0	5·1	0·9	9·2	6·5	2
	E	+50	3·9	0·6	6·5	4·7	2
	E	−50	8·3	1·1	14·2	10·1	3
	E	0	5·8	0·6	9·9	6·8	3
	E	+50	4·4	0·4	7·5	5·1	3

This ordinary differential equation with an accuracy up to a factor of $\frac{1}{4}$ in the right hand-side coincides with the equation available in [9].

To satisfy conditions (26), assume

$$\Theta(\xi,\eta,\tau)\big|_{\tau<0} = 0, \quad T_1^*(\xi,\eta,\tau)\big|_{\tau<0} = T_e^*(\xi,\eta). \tag{38}$$

Then, taking into account (36) and (38) we obtain from (26)

$$\left.\frac{\mathrm{d}N}{\mathrm{d}\eta}\right|_{\eta=0} = 0, \quad N\big|_{\eta=\infty} = 1. \tag{39}$$

The solution to boundary-value problem (37) and (39) has the form

$$T_e^*(\xi,\eta) \overset{\text{def}}{=} N(\eta)$$

$$= 1 + (\gamma-1)M_\infty^2\, Pr \int_\theta^\infty [f''(\alpha)]^{Pr} \int_\eta^\alpha [f''(\beta)]^{2-Pr} \times \mathrm{d}\beta\, \mathrm{d}\alpha. \tag{40}$$

Thus, solution (40) is the initial temperature field in the conjugated problem considered.

7. STRUCTURE OF GENERAL SOLUTION

Pass to finding $\Theta = \Theta(\xi,\eta,\tau)$ from the equation

$$\frac{\partial^2\Theta}{\partial\eta^2} + Prf\frac{\partial\Theta}{\partial\eta} = Prf'\xi\frac{\partial\Theta}{\partial\xi} + B\xi^2\frac{\partial\Theta}{\partial\tau}. \tag{41}$$

For this purpose, first of all, use the following conditions

$$\left.\frac{\partial\Theta}{\partial\xi}\right|_{\xi=0} = 0, \quad \Theta(\xi,\eta,\tau)\big|_{\eta\to\infty} = 0,$$
$$\Theta(\xi,\eta,\tau)\big|_{\tau<0} = 0. \tag{42}$$

Conjugation conditions (33) and (34) with regard for (35) and (36), (38) and (39) take the form

$$\Theta\big|_{\eta=0} = \Theta_s\big|_{\substack{\zeta=0\\ z=\xi^2}}, \qquad Mh\frac{1}{\xi}\left.\frac{\partial\Theta}{\partial\eta}\right|_{\eta=0} = \left.\frac{\partial\Theta_s}{\partial\eta}\right|_{\substack{\zeta=0\\ z=\xi^2}}$$

The solution of equation (41) is sought in the form $\Theta = \sum_{i=1}^{2}\Theta^{(i)}$,

$$\Theta^{(i)} = \sum_k Y_k^{(i)}(\eta,\tau)\exp\left\{\frac{Pr(F_k^{(i)}-F)}{2}\right\}\xi^{2(\rho^{(i)}+k+1)} + \sum_e \Phi_e^{(i)}(\eta,\tau)\exp\left\{\frac{Pr(P_e^{(i)}-F)}{2}\right\} \times \xi^{2(\rho^{(i)}+l+\frac{1}{2})}, \tag{43}$$

where

$$F_k^{(i)} = \int_0^\eta f_k^{(i)}(\eta)\,\mathrm{d}\eta, \quad P_l^{(i)} = \int_0^\eta p_l^{(i)}(\eta)\,\mathrm{d}\eta,$$

$$F = \int_0^\eta f(\eta)\,\mathrm{d}\eta, \quad (\rho^{(1)} = \rho, \rho^{(2)} = 0).$$

(The limits of summation are given below.)

The functions $f_k^{(i)}(\eta)$ and $p_l^{(i)}(\eta)$ are determined from the equation†

† Equation (44) is analysed in Appendix 1.

$$q_m^{(i)\prime} + \frac{Pr}{2} q_m^{(i)2} = (2\mu_m^{(i)} + 1)f' + \frac{Pr}{2} f^2,$$

$$q_m^{(i)}(0) = 0, \tag{44}$$

where if $q_m^{(i)} \stackrel{\text{def}}{=} f_m^{(i)}$, then $\mu_m^{(i)} \stackrel{\text{def}}{=} 2(\rho^{(i)} + m + 1)$, (45)

and if $q_m^{(i)} \stackrel{\text{def}}{=} p_m^{(i)}$, then $\mu_m^{(i)} \stackrel{\text{def}}{=} 2(\rho^{(i)} + m + \frac{1}{2})$.

Substitution of (43) into equation (41) leads to the system of equations

$$\frac{\partial^2 Y_k^{(i)}}{\partial \eta^2} + Pr\, f_k^{(i)} \frac{\partial Y_k^{(i)}}{\partial \eta} = B \frac{\partial Y_{k-1}^{(i)}}{\partial \tau} \exp\left\{\frac{Pr}{2}(F_{k-1}^{(i)} - F_k^{(i)})\right\}, \tag{46}$$

$$Y_k^{(i)}(\eta, \tau) \to 0 \quad \text{at } \eta \to \infty \tag{47}$$

$$\frac{\partial^2 \Phi_l^{(i)}}{\partial \eta^2} + Pr\, p_l^{(i)} \frac{\partial \Phi_l^{(i)}}{\partial \eta} = B \frac{\partial \Phi_{l-1}^{(i)}}{\partial \tau} \exp\left\{\frac{Pr}{2}(P_{l-1}^{(i)} - P_l^{(i)})\right\}, \tag{48}$$

$$\Phi_l^{(i)}(\eta, \tau) \to 0 \quad \text{at } \eta \to \infty. \tag{49}$$

From condition (42) it follows that

$k = -1,0,1\ldots,\quad l = 1,2,3,\ldots$ at $\rho = 0$;

$k = 0,1,2,\ldots,\quad l = -1,0,1,\ldots$ at $\rho = \frac{1}{2}$;

$k = -1,0,1,\ldots,\quad l = 0,1,2,\ldots$ at $\frac{1}{2} < \rho < 1$;

$k = 0,1,2,\ldots,\quad l = 0,1,2,\ldots$ at $0 < \rho < \frac{1}{2}$. (49′)

From (49′) it follows that $k \geqslant -1$, $l \geqslant -1$ for any $\rho \in [0,1)$. The solution of equations (46) and (48) is sought in the form

$$Y_k^{(i)}(\eta, \tau) = \sum_{j=0}^{k+1} (-B)^j \frac{d^j \bar{y}_{k-j}^{(i)}}{d\tau^j} \psi_{jk}^{(i)}(\eta), \tag{50}$$

$$\Phi_l^{(i)}(\eta, \tau) = \sum_{j=0}^{l+1} (-B)^j \frac{d^j \varphi_{l-j}^{(i)}}{d\tau^j} \kappa_{jl}^{(i)}(\eta), \tag{51}$$

where

$$y_k^{(i)}(\tau) = Y_k^{(i)}\big|_{\eta=0}, \qquad \bar{y}_k^{(i)}(\tau) = \frac{\partial Y_k^{(i)}}{\partial \eta}\bigg|_{\eta=0}$$

$$\varphi_l^{(i)}(\tau) = \Phi_l^{(i)}\big|_{\eta=0}, \qquad \bar{\varphi}_l^{(i)}(\tau) = \frac{\partial \Phi^{(i)}}{\partial \eta}\bigg|_{\eta=0}. \tag{52}$$

Substitution of (50) and (51) into equations (46) and (48) and separation of variables reduce to the system of the equations

$$\frac{d^2\psi_{jk}^{(i)}}{d\eta^2} + Prf_k^{(i)} \frac{d\psi_{jk}^{(i)}}{d\eta} = -(1 - \delta_{0,j}) \times \exp\left(\frac{Pr}{2}(F_{k-1}^{(i)} - F_k^{(i)})\right) \psi_{j-1,k-1}^{(i)}(\eta),$$

$$\psi_{j,k}^{\prime(i)}(0) = \delta_{0,j}, \quad \psi_{j,k}^{(i)}(\infty) = 0,$$

$$\delta_{0,j} = \begin{cases} 1 \text{ at } j = 0, \\ 0 \text{ at } j = 1, 2, \ldots k + 1. \end{cases} \tag{53}$$

$$\frac{d^2\varkappa_{jl}^{(i)}}{d\eta^2} + Prp_l^{(i)} \frac{d\varkappa_{jl}^{(i)}}{d\eta} = -(1 - \delta_{0,j})\varkappa_{j-1,l-1}^{(i)} \times \exp\left(\frac{Pr}{2}(P_{l-1}^{(i)} - P_l^{(i)})\right),$$

$$\varkappa_{j,l}^{(i)}(0) = \delta_{0j}, \quad \varkappa_{j,l}^{(i)}(\infty) = 0,$$

$$j = 1, 2, \ldots, l + 1. \tag{54}$$

From (50) and (51) it follows that

$$Y_{k_{\min}}^{(i)}(\eta, \tau) = \frac{\partial Y_{k_{\min}}^{(i)}}{\partial \eta}\bigg|_{\eta=0} \psi_{0,k_{\min}}^{(i)}(\eta),$$

$$\Phi_{l_{\min}}^{(i)}(\eta, \tau) = \frac{\partial \Phi_{l_{\min}}^{(i)}}{\partial \eta}\bigg|_{\eta=0} \varkappa_{0,l_{\min}}^{(i)}(\eta), \tag{54′}$$

where $k_{\min}$ and $l_{\min}$ are initial numbers.

Let us average heat conduction equation (27) and conditions (28), (29) and (31) over the dimensionless plate thickness $h = H/L$

$$\frac{\partial \tilde{\Theta}_s}{\partial \tau} = \frac{\partial^2 \tilde{\Theta}_s}{\partial z^2} + \frac{1}{h}\left[\frac{\partial \Theta_s}{\partial \zeta}\bigg|_{\zeta=0} - \frac{\partial \Theta}{\partial \zeta}\bigg|_{\zeta=-h}\right] + \tilde{Q}(z, \tau), \tag{55}$$

† Systems (53) and (54) are considered in Appendix II.

where

$$\tilde{\Theta}_s = \frac{1}{h}\int_{-h}^{0} \Theta_s(z,\zeta,\tau)\,d\zeta,$$

$$\tilde{Q} = \frac{1}{h}\int_{-h}^{0} Q(z,\zeta,\tau)\,d\zeta. \quad (56)$$

Since $h \ll 1$, it is possible to assume

$$\tilde{\Theta}_s = \Theta_s|_{\zeta=0}. \quad (57)$$

So using conditions (30), (33), (34) and (57), transform equation (55) and averaged conditions (28), (29) and (31) into the form:

$$\frac{\partial\Theta|_{\eta=0}}{\partial\tau} = \frac{\partial^2\Theta|_{\eta=0}}{\partial z^2} + \frac{M}{\sqrt{z}}\frac{\partial\Theta}{\partial\eta}\bigg|_{\eta=0} + R(z,\tau), \quad (58)$$

where

$$R(z,\tau) = \tilde{Q} - \frac{1}{h}F = \sum_{k=0}^{\infty} R_k(\tau)\,z^{\rho+k} \quad (59)$$

is in reality the new known source.

$$\left(\frac{\partial\Theta|_{\eta=0}}{\partial z}\right)_{z=0} = 0, \quad (60)$$

$$\left(\frac{\partial\Theta|_{\eta=0}}{\partial z}\right)_{z=\infty} = 0, \quad (61)$$

$$\Theta_{\substack{\eta=0\\ \tau<0}} = 0. \quad (62)$$

The solution of Θ for thermal problem (41), (42) was earlier presented as $\Theta = \Theta^{(1)} + \Theta^{(2)}$, where $\Theta^{(i)} (i = 1, 2)$ satisfies equation (41) and conditions (42).

Designate

$$\Theta|_{\eta=0,z=0} = A(\tau) \quad (63)$$

$\overset{(1)}{\Theta}|_{\eta=0}$ corresponds to equation (58), conditions (60) and (62) and the additional condition

$$(\overset{(1)}{\Theta}|_{\eta=0})|_{z=0} = 0 \quad (64)$$

$\Theta^{(2)}|_{\eta=0}$ satisfies the equation

$$\frac{\partial\Theta^{(2)}|_{\eta=0}}{\partial\tau} = \frac{\partial^2\Theta^{(2)}|_{\eta=0}}{\partial z^2} + \frac{M}{\sqrt{z}}\left(\frac{\partial\Theta^{(2)}}{\partial\eta}\right)\bigg|_{\eta=0}, \quad (65)$$

conditions (60) and (62) and the additional condition

$$(\Theta^{(2)}|_{\eta=0})|_{z=0} = A(\tau). \quad (66)$$

The function $A(\tau)$ will be found later on from condition (61), which the function should satisfy

$$\Theta|_{\eta=0} = \Theta^{(1)}|_{\eta=0} + \Theta^{(2)}|_{\eta=0}. \quad (67)$$

Pass to determing $\Theta^{(1)}$. For this purpose it is left to find $\bar{y}_k^{(1)}$, $\varphi_k^{(1)}$ (See (43)–(54').) Because at $\eta = 0$ $z = \xi^2$, then from (43) we have

$$\Theta^{(1)}|_{\eta=0} = \sum_k y_k^{(1)}(\tau)\,z^{\rho+k+1} + \sum_l \varphi_l^{(1)}(\tau)\,z^{\rho+l+\frac{1}{2}}. \quad (68)$$

Substitute (43) and (68) into equation (58) and equate the expressions at the same powers of z. Then we arrive at

$$\frac{dy_k^{(1)}(\tau)}{d\tau} = (\rho+k+3)(\rho+k+2)\,y_{k+2}^{(1)}(\tau) + R_{k+1}(\tau) + M\bar{\varphi}_{k+1}^{(1)}(\tau), \quad (69)$$

$$\frac{d\varphi_k^{(1)}(\tau)}{d\tau} = (\rho+k+\tfrac{5}{2})(\rho+k+\tfrac{3}{2})\,\varphi_{k+2}^{(1)}(\tau) + M\bar{y}_k^{(1)}(\tau) \quad (k = -1, 0, 1, \ldots). \quad (70)$$

From (49'), (60) and (64) and relationship (54') it is seen, that†

$$y_k^{(1)}(\tau) \equiv 0 \text{ for } k < 1; \quad \varphi_k^{(1)}(\tau) \equiv 0 \text{ for } k < 3. \quad (71)$$

Write relations (70) for $k = 0, 2, 4, \ldots, 2(m-1)$ and sum them up, preliminarily by acting upon the ith line by the operator $\Gamma(\rho + 2i - \frac{1}{2})\,d^{m-i}/d\tau^{m-i}$. Make the same operations for $k = -1, 1, 3, \ldots, (2m-3)$ using the operator $\Gamma(\rho + 2i - \frac{3}{2})\,d^{m-i}/d\tau^{m-i}$. Then we have for even k

$$\varphi_{2m}^{(1)}(\tau) = -M\sum_{i=2}^{m}\frac{\Gamma(\rho+2i-\frac{1}{2})}{\Gamma(\rho+2m+\frac{3}{2})}\frac{d^{m-i}}{d\tau^{m-i}} \times \bar{y}_{2(i-1)}^{(1)}(\tau), \quad (72)$$

† Note that from (70) with regard for (71) at $k = -1, 0$ we have $\bar{y}_{-1}^{(1)}(\tau) \equiv 0$, $\bar{y}_0^{(1)}(\tau) \equiv 0$.

$$\varphi^{(1)}_{2m-1}(\tau) = -M\sum_{i=2}^{m}\frac{\Gamma(\rho+2i-\frac{3}{2})}{\Gamma(\rho+2m+\frac{1}{2})}\frac{\mathrm{d}^{m-i}}{\mathrm{d}\tau^{m-i}} \times \bar{y}^{(1)}_{2i-3}(\tau) \quad (m = 2,3,4,\ldots). \tag{73}$$

for odd k. Thus, $\varphi^{(1)}_k(\tau)$ are defined in terms of $\bar{y}^{(1)}_k(\tau)$. Use relation (69). Write down $\bar{\varphi}^{(1)}_k(\tau)$ and $y^{(1)}_k(\tau)$ from (50) and (51) with regard for (71)

$$y^{(1)}_k(\tau) = \sum_{j=0}^{k-1}(-B)^j\frac{\mathrm{d}^j\bar{y}^{(1)}_{k-j}}{\mathrm{d}\tau^j}\psi^{(1)}_{jk}(0), \tag{74}$$

$$\bar{\varphi}^{(1)}_k(\tau) = \sum_{j=0}^{k-3}(-B)^j\frac{\mathrm{d}^j\varphi^{(1)}_{k-j}}{\mathrm{d}\tau^j}\varkappa'^{(1)}_{jk}(0). \tag{75}$$

From (69), (72)–(75) $\bar{y}^{(1)}_k(\tau)$ are defined in terms of $R_k(\tau)$—the coefficients of the sources. Thus, $\varphi^{(1)}_k(\tau)$, $\bar{y}^{(1)}_k(\tau)$ and consequently $\Theta^{(1)}$ (see (43)–(54')) are found.

Now determine $\Theta^{(2)}$. For this purpose we need $\bar{y}^{(2)}_k(\tau)$ and $\varphi^{(2)}_k(\tau)$. (See (43)–(54').) As in case of $\Theta^{(1)}|_{\eta=0}$, the expression for $\Theta^{(2)}|_{\eta=0}$ is obtained from (43) assuming, however, $\rho = 0$, since $\Theta^{(2)}|_{\eta=0}$ satisfies equation (65) with no sources

$$\Theta^{(2)}|_{\eta=0} = \sum_{k=-1}^{\infty} y^{(2)}_k(\tau)\,z^{k+1} + \sum_{l=-1}^{\infty}\varphi^{(2)}_l(\tau)\,z^{\rho+\frac{1}{2}} \tag{76}$$

($z = \xi^2$ at $\eta = 0$; $k \geqslant -1, l \geqslant -1$ from (49')). When $\Theta^{(2)}|_{\eta=0}$ obeys condition (60), we have

$$y^{(2)}_0(\tau) \equiv 0, \quad \varphi^{(2)}_{-1}(\tau) = \varphi^{(2)}_0(\tau) \equiv 0. \tag{77}$$

Therefore, $\Theta^{(2)}|_{=0}$ assumes the form

$$\Theta^{(2)}|_{\eta=0} = y^{(2)}_{-1}(\tau) + \sum_{k=1}^{\infty} y^{(2)}_k(\tau)\,z^{k+1} + \sum_{l=1}^{\infty}\varphi^{(2)}_l(\tau)\,z^{l+\frac{1}{2}} \tag{78}$$

(from (66) and (78) it is seen that $y^{(2)}_{-1}(\tau) = A(\tau)$).

Before applying (78) to obtaining the recurrent relations, similar to (69) and (70), turn to the expression for the specific heat flux from the liquid to the plate $q = q_1 + q_2$,

$$q_1 = \frac{M'h\lambda_s}{\sqrt{z}}\frac{\partial\Theta^{(1)}}{\partial\eta}\bigg|_{\eta=0} = = M'h\lambda_s\Big[\sum_{k=1}^{\infty}\bar{y}^{(1)}_k(\tau)\,z^{\rho+k+\frac{1}{2}} + \sum_{l=3}^{\infty}\bar{\varphi}^{(1)}_l(\tau)\,z^{\rho+l}\Big],$$

$$q_2 = \frac{M'h\lambda_s}{\sqrt{z}}\frac{\partial\Theta^{(2)}}{\partial\eta}\bigg|_{\eta=0} = M'h\lambda_s\left[\frac{\bar{y}^{(2)}_{-1}(\tau)}{\sqrt{z}} + \sum_{k=0}^{\infty}\bar{y}^{(2)}_k(\tau)\,z^{k+\frac{1}{2}} + \sum_{l=1}^{\infty}\bar{\varphi}^{(2)}_l(\tau)\,z^l\right], \tag{79}$$

where

$$M' = -\frac{T_\infty}{L}M.$$

It is hence seen that the series for $q_1(z,\tau)$ should be continued to the left up to $z = 0$. At the point $z = 0$ the series for $q_2(z,\tau)$ has an integrable peculiarity

$$q_2 = M'h\lambda_s\bar{y}^{(2)}_{-1}(\tau)\,z^{-\frac{1}{2}} + 0(z) \quad \text{at} \quad z \to 0,$$

$$\int_0^{z_1} q_2(z,\tau)\,\mathrm{d}z = \int_0^{z_1} M'h\lambda_s z^{-\frac{1}{2}} \times \frac{\partial\Theta^{(2)}}{\partial\eta}\bigg|_{\eta=0}\mathrm{d}z < \infty\ (z_1 > 0).$$

It should be noted that equation (65) incorporates the expression

$$\frac{M}{\sqrt{z}}\frac{\partial\Theta^{(2)}}{\partial\eta}\bigg|_{\eta=0} = -\frac{Lq_2(z,\tau)}{T_\infty h\lambda_s} \to \infty \text{ at } z \to 0.$$

Therefore, to obtain the relations similar to (69) and (70) (written, however, for $z > z_0$), use will be made of (65), (78) and (79)

$$\frac{\mathrm{d}y^{(2)}_k(\tau)}{\mathrm{d}\tau} = (k+3)(k+2)\,y^{(2)}_{k+2} + M\bar{\varphi}^{(2)}_{k+1}, \tag{80}$$

† $Re = \frac{U_\infty L}{v_\infty}$, $Re_{x_0} = \frac{U_\infty x_0}{v_\infty} = \frac{x_0}{L}Re = z_0 \cdot Re.$

Since at $Re_{x_0} \approx 10$ the boundary layer approximation comes into force, then, $z_0 \sim 10/Re$.

$$\frac{d\varphi_k^{(2)}(\tau)}{d\tau} = (k + \tfrac{5}{2})(k + \tfrac{3}{2})\varphi_{k+2}^{(2)} + M\bar{y}_k^{(2)}. \qquad (81)$$

Solution (81) similar to solution (70) allows $\varphi_k^{(2)}(\tau)$ to be expressed in terms of $\bar{y}_k^{(2)}(\tau)$

$$\varphi_{2m}^{(2)} = -M\sum_{i=1}^{m}\frac{\Gamma(2i-\frac{1}{2})}{\Gamma(2m+\frac{3}{2})} \times \frac{d^{m-i}}{d\tau^{m-i}}\bar{y}_{2(i-1)}^{(2)}, \qquad (82)$$

$$\varphi_{2m-1}^{(2)} = -M\sum_{i=1}^{m}\frac{\Gamma(2i-\frac{3}{2})}{\Gamma(2m+\frac{1}{2})} \times \frac{d^{m-i}}{d\tau^{m-i}}\bar{y}_{2i-3}^{(2)}(\tau). \qquad (83)$$

The functions $\bar{y}_k^{(2)}$ $(k = 0, 1, 2, \ldots)$ may be expressed in terms of the function $\bar{y}_{-1}^{(2)}(\tau)$ (which is unknown up to this moment) if expressions (82) and (83) together with those obtained from (50) and (51)

$$y_k^{(2)}(\tau) = \sum_{j=0}^{k+1}(-B)^j\frac{d^j\bar{y}_{k-j}^{(2)}}{d\tau^j}\psi_{jk}^{(2)}(0), \qquad (84)$$

$$\varphi_k^{(2)}(\tau) = \sum_{j=0}^{k-1}(-B)^j\frac{d^j\varphi_{k-j}^{(2)}}{d\tau^j}\varkappa'^{(2)}_{jk}(0) \qquad (85)$$

are included into recurrent relation (80). It is left to find the function† $\bar{y}_{-1}^{(2)}(\tau)$, since the functions $\bar{y}_k^{(2)}(\tau)$ $(k = 0, 1, 2, \ldots)$, $\varphi_k^{(2)}(\tau)$ and, consequently, $\Theta^{(2)}$ are found (see (43)–(54′)). Therefore, pass again to equation (58), whose solution under the initial and boundary conditions (60)–(62) is

$$\Theta|_{\eta=0} = \Theta^{(1)}|_{\eta=0} + \Theta^{(2)}|_{\eta=0}. \qquad (86)$$

The solution of one-dimensional boundary-value problem (58), (60)–(62) assumes the form

$$\Theta|_{\eta=0} = \int_0^\infty \frac{1}{2\sqrt{(\pi\tau)}} \times \left[\exp\left(-\frac{(z-z')^2}{4\tau}\right) + \exp\left(-\frac{(z+z')^2}{4\tau}\right)\right] * \left[\frac{M}{\sqrt{z'}}\frac{\partial\Theta(z',\eta,\tau)}{\partial\eta}\bigg|_{\eta=0} + R(z',\tau)\right]dz'. \qquad (87)$$

(* is the convolution.)

From (87) with regard for (63) and the footnote we have

$$\psi_{0,-1}^{(2)}\bar{y}_{-1}^{(2)}(\tau) = \int_0^\infty \frac{\exp(-z'^2/4\tau)}{\sqrt{(\pi\tau)}} * \left[\frac{M}{\sqrt{z'}}\frac{\partial\Theta(z',\eta,\tau)}{\partial\eta}\bigg|_{\eta=0} + R(z',\tau)\right]dz'. \qquad (88)$$

† In this case the temperature of the leading edge of a plate is to be found since from (84) it follows that

$$A(\tau) = y_{-1}^{(2)}(\tau) = \psi_{0,-1}^{(2)}(0)\,\bar{y}_{-1}^{(2)}(\tau).$$

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TRANSFERT THERMIQUE CONJUGUE NON PERMANENT ENTRE UNE SURFACE SEMI-INFINIE ET UN ECOULEMENT INCIDENT DE FLUIDE COMPRESSIBLE—I. REDUCTION A LA RELATION INTEGRALE

Résumé— La solution analytique du problème conjugué du transfert thermique non permanent dans une plaque semi-infinie avec sources sous la forme de séries entières généralisées est obtenue dans l'espace des fonctions généralisées (au sens de Sobolev–Schwartz).

On a trouvé le paramètre adimensionnel $B = (1/Re)(a_s/Ca_\infty)$ ayant un sens physique qui permet de considérer le problème comme non permanent ou quasi-stationnaire dépendant du couple "obstacle-liquide".

INSTATIONÄRE KONJUGIERTE WÄRMEÜBERTRAGUNG ZWISCHEN EINER HALBUNENDLICHEN OBERFLÄCHE UND EINEM ANSTRÖMENDEN KOMPRESSIBLEN FLUID. ZURÜCKFÜHRUNG AUF DIE INTEGRALGLEICHUNG

Zusammenfassung— Die analytische Lösung des konjugierten instationären Wärmeübertragungsproblems an einer halbunendlichen Platte mit Wärmequellen wurde in Form verallgemeinerter Potenzreihen aus dem Raum verallgemeinerter Funktionen (im Sobolev–Schwartz'schen Sinne) erhalten. Es wurde ein dimensionsloser Parameter $B = [1/Re \cdot a_s/Ca_\infty]$ mit physikalischer Bedeutung gefunden, der eine Betrachtung des instationären und quasistationären Problems ermöglicht und nur von der Paarung "Körper-Flüssigkeit" abhängt.

НЕСТАЦИОНАРНЫЙ СОПРЯЖЕННЫЙ ТЕПЛООБМЕН ПОЛУБЕСКОНЕЧНОЙ ПОВЕРХНОСТИ С НАТЕКАЮЩИМ ПОТОКОМ СЖИМАЕМОЙ ЖИДКОСТИ I. СВЕДЕНИЕ К ИНТЕГРАЛЬНОМУ СООТНОШЕНИЮ

Аннотация—В пространстве обобщенных функций в смысле Соболева-Шварца получено аналитическое решение задачи сопряженного нестационарного т плообмена полубесконечной пластины с источниками в виде обобщенного степеного ряда.

Выделен обладающий физическим смыслом безразмерный параметр

$$B = \left[\frac{1}{\mathrm{Re}} \cdot \frac{a_s}{Ca_\infty}\right],$$

который позволяет рассматривать задачу как нестационарную или квазистационарную в зависимости от пары «тело–жидкость».